P P SAVANI UNIVERSITY

Second Semester of B. Tech. Examination

May 2019

SESH1020 Linear Algebra & Vector Calculus

14.05.2019, Tuesday **Time: 12:30 p.m. To 03:00 p.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1 Answer the following. (Any Five) **[05]**

(i) Show that matrix multiplication is not commutative AB ≠BA in general.

(ii) Find the eigenvalues of $A = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix}$.

(iii) Define Linear Independence of vector.

(iv) What condition must be satisfied for being linear transformation?

(v) Define Kernel of Linear Transformation.

(vi) "Two forces applied at the same time & at the same spot will have same effect of certain single force", is example of which vector operation?

Q - 2 (a) Answer the following. (Any Two) **[04]**

1. Find the rank of the matrix $A = \begin{bmatrix} 0 & 6 & 7 \\ -5 & 4 & 2 \\ 1 & -2 & 0 \end{bmatrix}$.

2. Prove or disapprove that the set $W = \{(a, 2a, a+1) | a \in R\}$ is not a vector sub-space of R^3.

3. Determine whether the vector $v = (-5, 11, -7)$ is a linear combination of vectors $v_1 = (1, -2, 2), v_2 = (0, 5, 5)$ and $v_3 = (2, 0, 8)$.

Q - 2 (b) Answer the following. (Any Two) **[06]**

1. Prove that the matrix $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$ is a Hermitian & iA is Skew Hermitian.

2. State Cauchy-Schwarz inequality in R^n. Hence, verify the same & also find $\cos\theta$ for the vectors $u_1 = (0, -2, 2, 1), u_2 = (-1, -1, 1, 1)$.

3. Find the value of a for which the following system of equation over ℝ is consistent. $x_1 + 3x_2 - x_3 = 1, 2x_1 + 7x_2 + ax_3 = 3, x_1 + ax_2 - 7x_3 = 0$.

Q - 3 Answer the following. (Any Three) **[05]**

(i) Determine algebraic & geometric multiplicity of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(ii) Find the characteristic equation of the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ & hence, find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$.

(iii) Find basis for the row & column space of $A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 1 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$.

(iv) Consider the basis $S=\{v_1,v_2\}$ for R^2, where $v_1=\begin{bmatrix}1\\1\end{bmatrix}$ & $v_2=\begin{bmatrix}2\\3\end{bmatrix}$. Let $T:R^2 \rightarrow P_2$ be the linear transformation such that $T(v_1)=2-3x+x^2$, $T(v_2)=1-x^2$.

Find $T\begin{bmatrix}a\\b\end{bmatrix}$ & then find $T\begin{bmatrix}-1\\2\end{bmatrix}$.

SECTION – II

Q - 1 Answer the following. (Any Five) **[05]**

(i) For a vectors u & v, $||u+v||^2 - ||u-v||^2$ is ______

a) < u, v > b) 2 < u, v > c) 3 < u, v > d) 4 < u, v >

(ii) The angle between $u=(1,0,0)$ & $v=(0,1,0)$ is ______°

a) 90 b) 45 c) 60 d) 0

(iii) The arc length of y = 5 from x=0 to x = 16 is

a) 5 b) 0 c) 16 d) 4

(iv) A vector point function is called a solenoidal vector function if

a) $div\ V = 0$ b) $div\ V > 0$ c) $div\ V < 0$ d) $div\ V \neq 0$

(v) For the Stokes's theorem direction of curve C is

a) Clockwise b) Anticlockwise c) Both d) None of these

(vi) The line integral for the vector field $F = x^2 i - xy\, j$ from the origin O to the point P(1 ,1) is

a) 2 b) 1 c) 0 d) $\frac{1}{12}$

Q - 2 (a) Answer the following. (Any Two) **[04]**

1. Determine whether there exists scalar k & p such that vectors $u=(2,k,6)$, $v=(p,5,3)$ & $w=(1,2,3)$ are mutually orthogonal with respect to the Euclidean inner product.
2. If $\phi = x^2+y^2+z^2-8$ then find $grad\ \phi$ at the point (2,0,2).
3. State Green & Stokes's Theorem.

Q - 2 (b) Answer the following. (Any Two) **[06]**

1. Calculate curl $\bar{F}$ if $\bar{F} = xyz\, i + 3x^2y\, j + (xz^2 - y^2z)k$ at the point $(1,-1,1)$.
2. If S is any closed surface enclosing volume V & $\bar{F} = ax\, i + by\, j + cz\, k$ then prove that $\iint_s \bar{F}.\hat{n}dS = (a+b+c)V$.
3. Show that the set of vectors $u_1 = \left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right)$, $u_2 = \left(\frac{-1}{2},\frac{1}{2},0\right)$ & $u_3 = \left(\frac{1}{3},\frac{1}{3},\frac{-2}{3}\right)$ is orthogonal with respect to the Euclidean inner product on R³ & then convert it to an orthonormal set by normalizing the vectors.

Q - 3 Answer the following. (Any Three) **[05]**

(i) Find the least square solution of the linear system Ax = b & find the orthogonal projection of b on the column space of A, where A = $\begin{bmatrix}1 & -1\\3 & 2\\-2 & 4\end{bmatrix}$ & b = $\begin{bmatrix}4\\1\\3\end{bmatrix}$.

(ii) Define Directional derivative & find directional derivative of $\phi = 4xz^2 + x^2yz$ at (1,-2,1) in the direction of $2i+3j+4k$.

(iii) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal & irrotational.

(iv) Evaluate $\iiint_v \nabla\bar{F}\, dV$ if $\bar{F} = x^2i + y^2j + z^2k$ and V is the volume of the region enclosed by the cube $x=0, x=1, y=0, y=1, z=0, z=1$.

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